Digital Image Processing

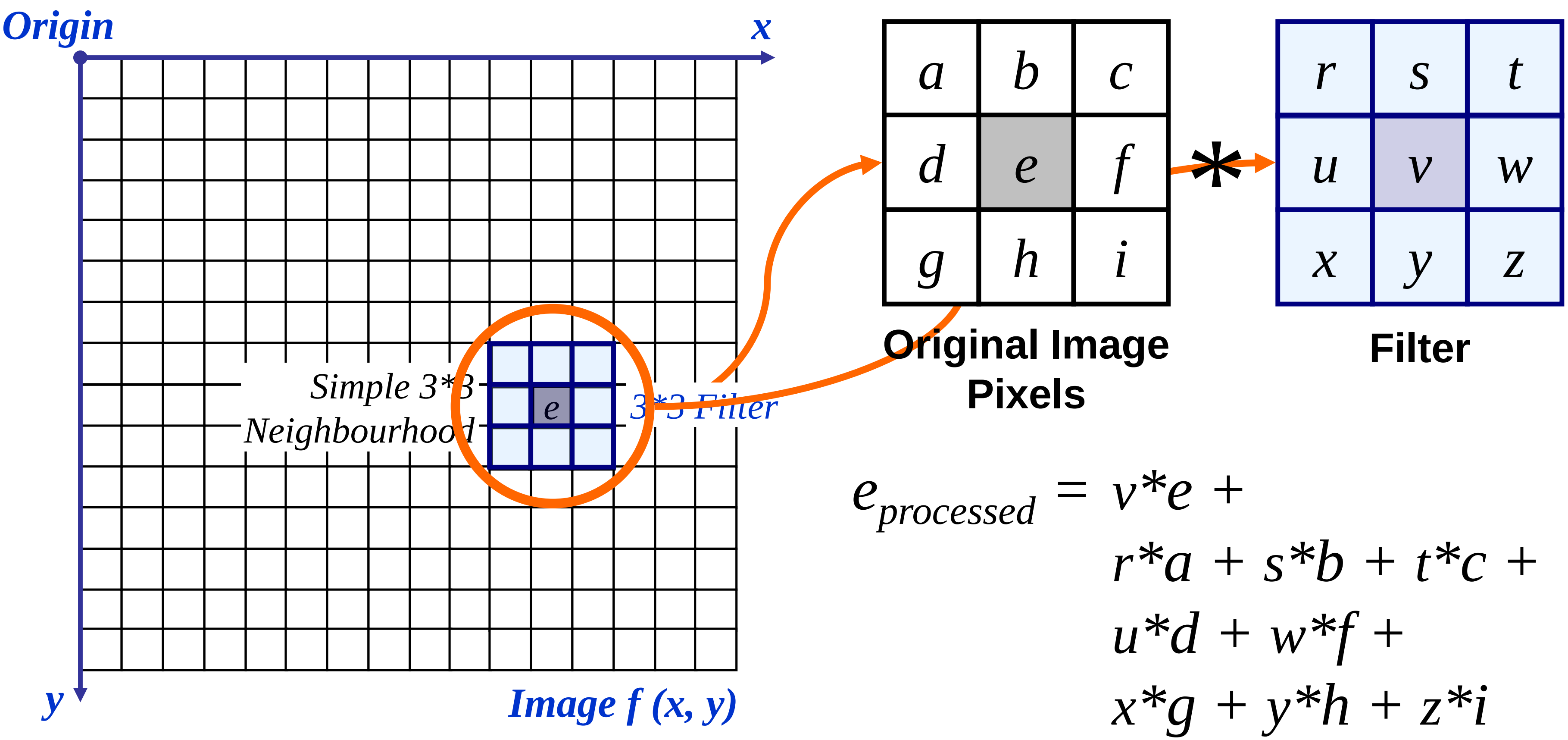
Image Enhancement
(Spatial Filtering 2)

Contents

In this lecture we will look at more spatial filtering techniques

- Spatial filtering refresher
- Sharpening filters
  - 1st derivative filters
  - 2nd derivative filters
- Combining filtering techniques

Spatial Filtering Refresher



$$
\begin{aligned}
e_{processed} = \; & v*e + \\
& r*a + s*b + t*c + \\
& u*d + w*f + \\
& x*g + y*h + z*i
\end{aligned}
$$

The above is repeated for every pixel in the original image to generate the smoothed image

Sharpening Spatial Filters

Previously we have looked at smoothing filters which remove fine detail

Sharpening spatial filters seek to highlight fine detail

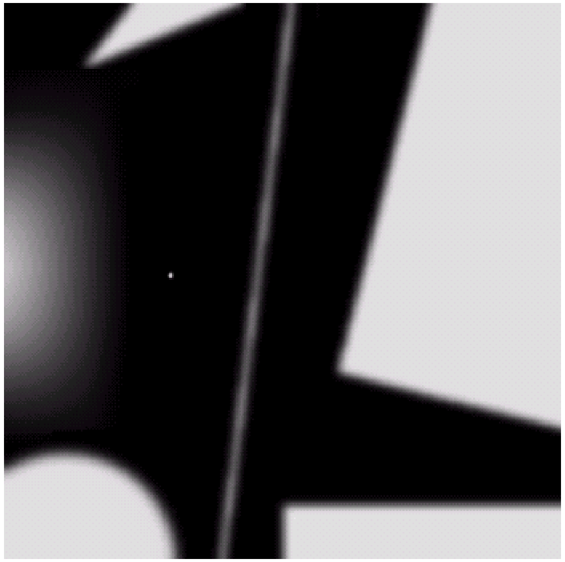
- Remove blurring from images
- Highlight edges

Sharpening filters are based on *spatial differentiation*

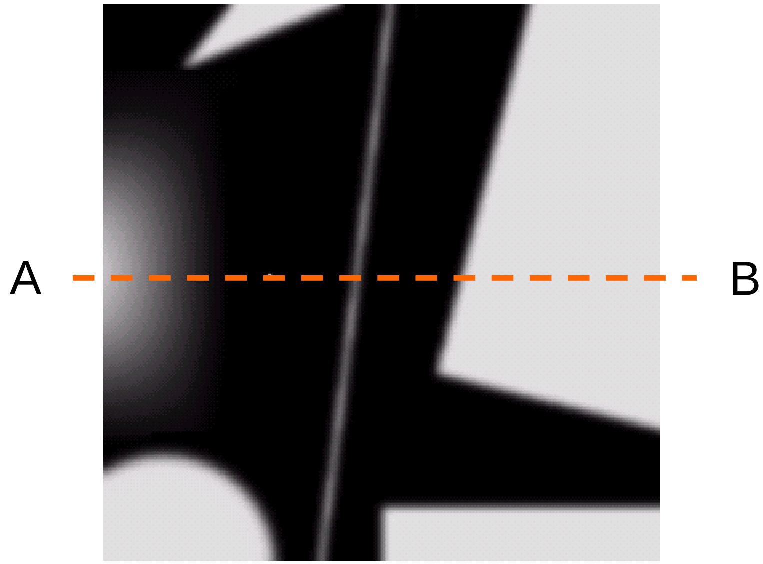
Spatial Differentiation

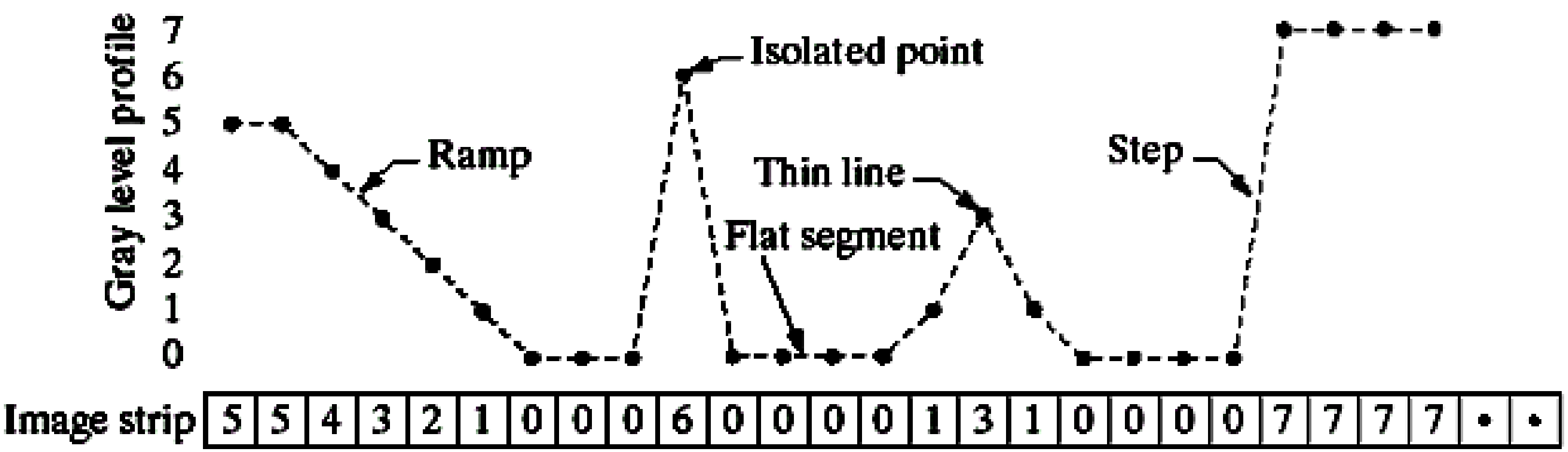
Differentiation measures the *rate of change* of a function

Let's consider a simple 1 dimensional example



Spatial Differentiation





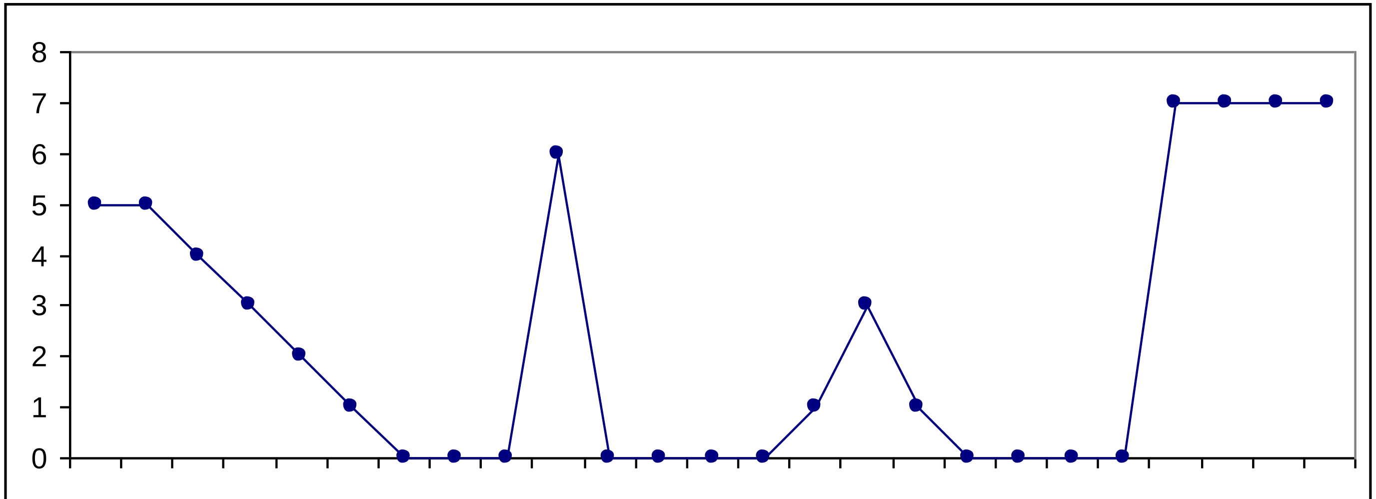
1^{st} Derivative

The formula for the 1^{st} derivative of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

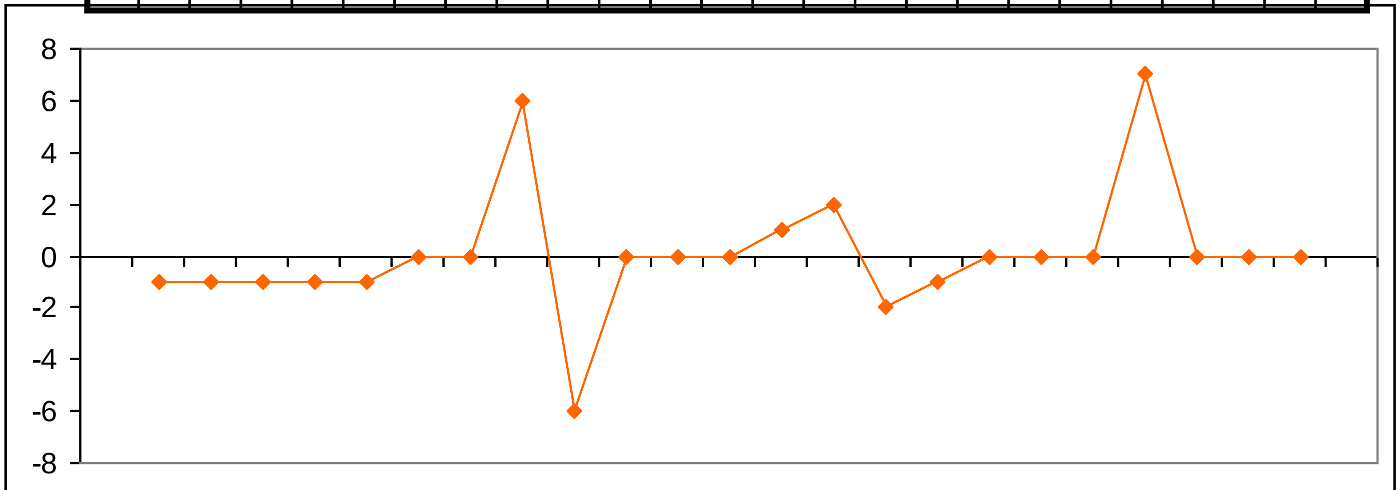
It's just the difference between subsequent values and measures the rate of change of the function

1^{st} Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	0	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	



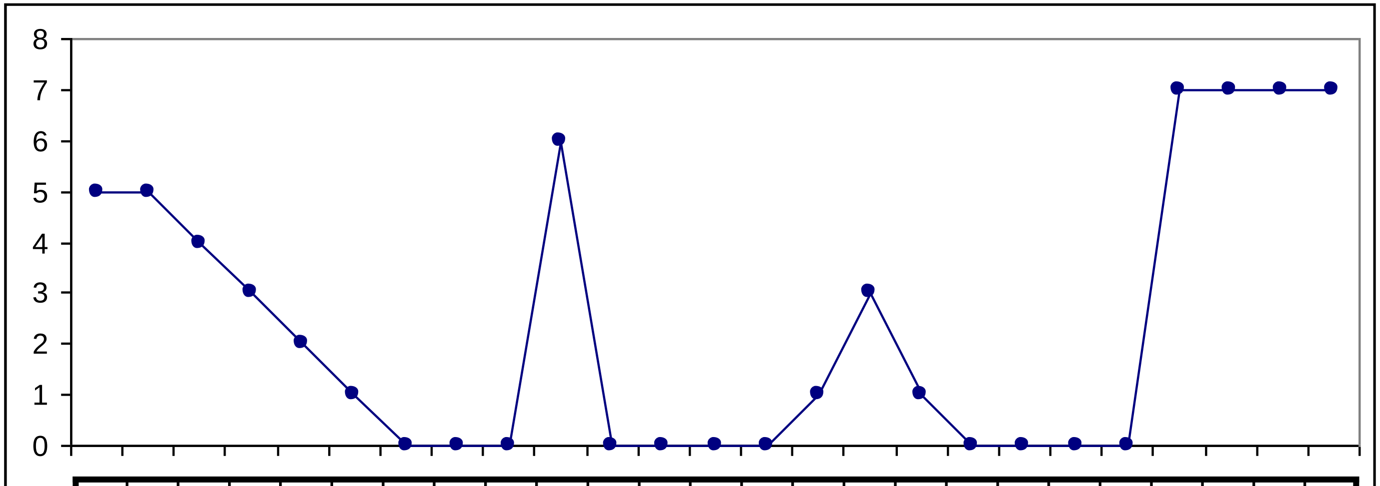
2nd Derivative

The formula for the 2nd derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

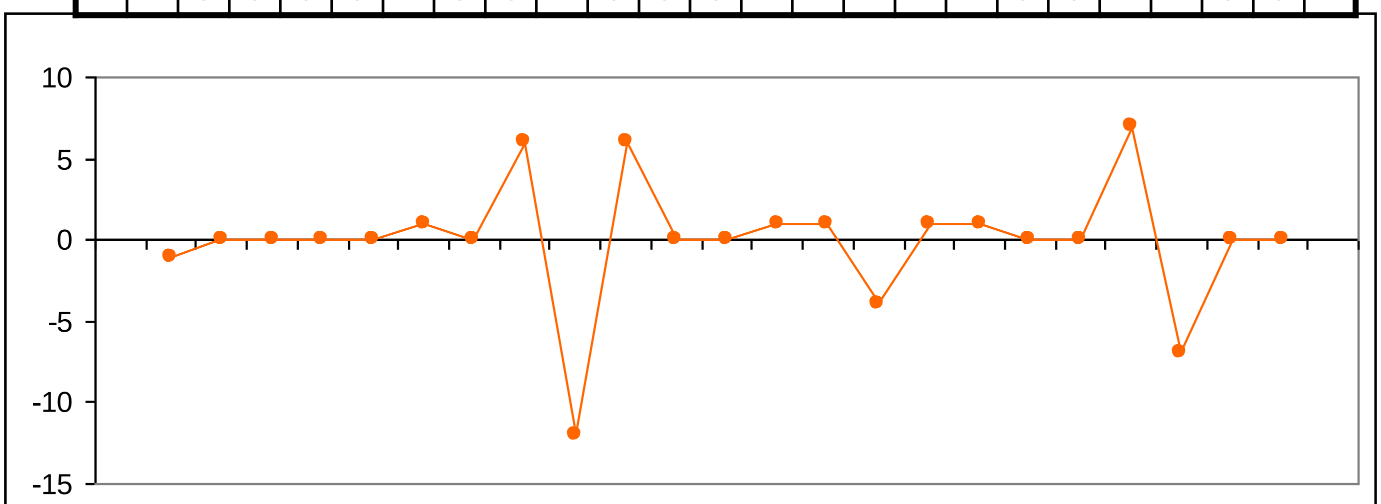
Simply takes into account the values both before and after the current value

2^{nd} Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



Using Second Derivatives For Image Enhancement

- The 2^{nd} derivative is more useful for image enhancement than the 1^{st} derivative
  - – Stronger response to fine detail
  - – Simpler implementation
  - – We will come back to the 1^{st} order derivative later on
- The first sharpening filter we will look at is the *Laplacian*
  - – Isotropic
  - – One of the simplest sharpening filters
  - – We will look at a digital implementation

The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 1st order derivative in the x direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the y direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

The Laplacian (cont…)

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

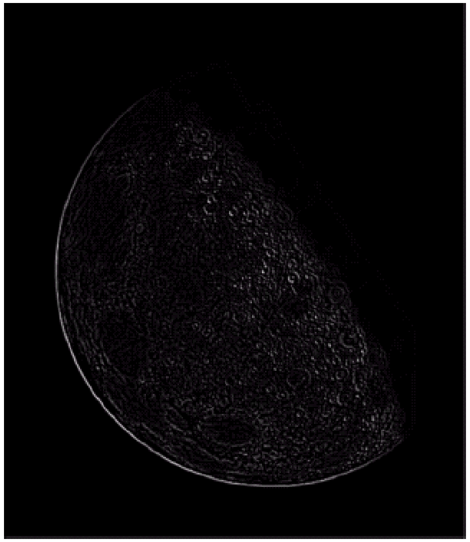
We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

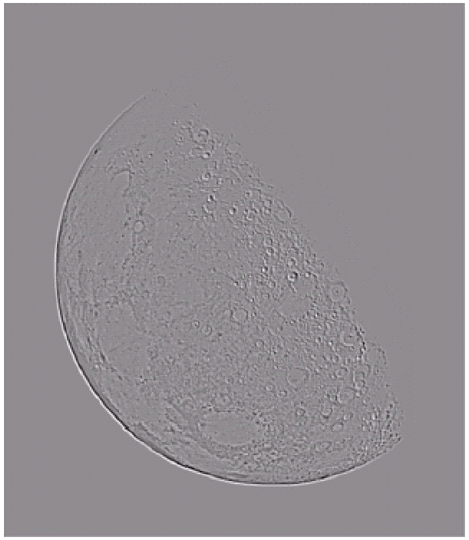
The Laplacian (cont…)

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original Image



Laplacian Filtered Image



Laplacian Filtered Image Scaled for Display

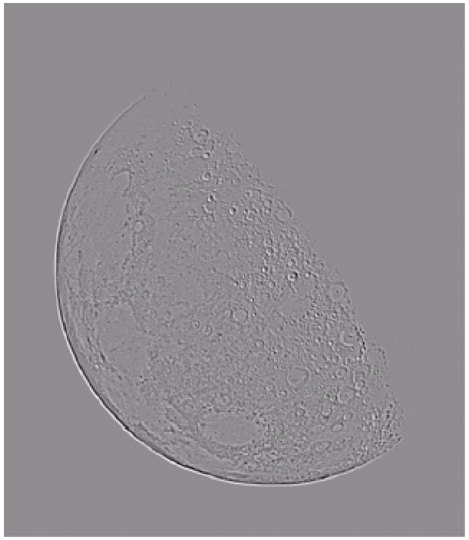
But That Is Not Very Enhanced!

The result of a Laplacian filtering is not an enhanced image

We have to do more work in order to get our final image

Subtract the Laplacian result from the original image to generate our final sharpened enhanced image

$$g(x,y) = f(x,y) - \nabla^2 f$$

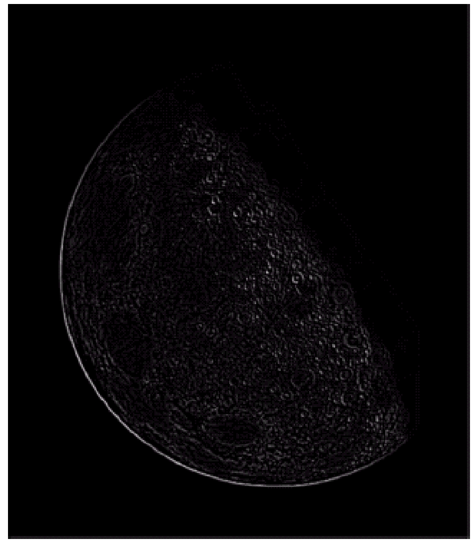


Laplacian Filtered Image Scaled for Display

Laplacian Image Enhancement

Original
Image

-



Laplacian
Filtered Image

=

Sharpened
Image

In the final sharpened image edges and fine detail are much more obvious

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Laplacian Image Enhancement

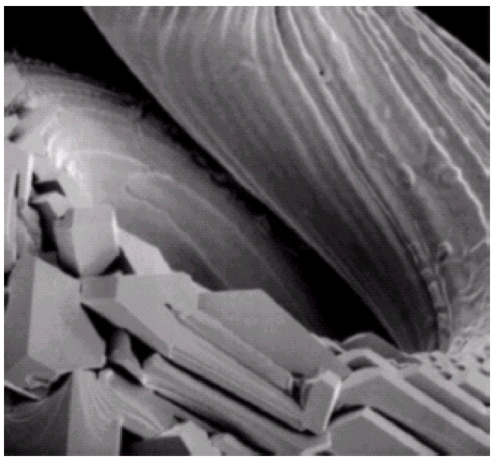
Simplified Image Enhancement

The entire enhancement can be combined into a single filtering operation

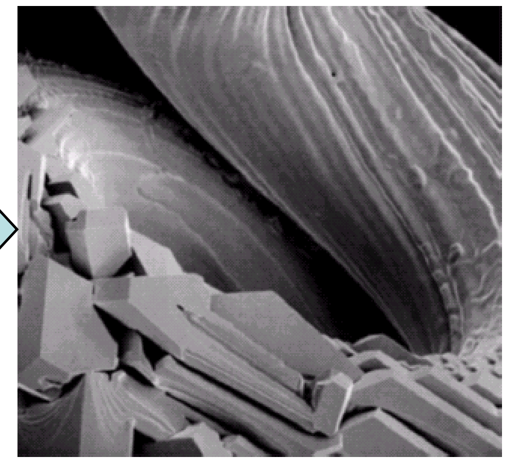
$$
\begin{aligned}
g(x, y) = f(x, y) - \nabla^2 f \\
= f(x, y) - [f(x+1, y) + f(x-1, y) \\
+ f(x, y+1) + f(x, y-1) \\
- 4f(x, y)] \\
= 5f(x, y) - f(x+1, y) - f(x-1, y) \\
- f(x, y+1) - f(x, y-1)
\end{aligned}
$$

Simplified Image Enhancement (cont…)

This gives us a new filter which does the whole job for us in one step

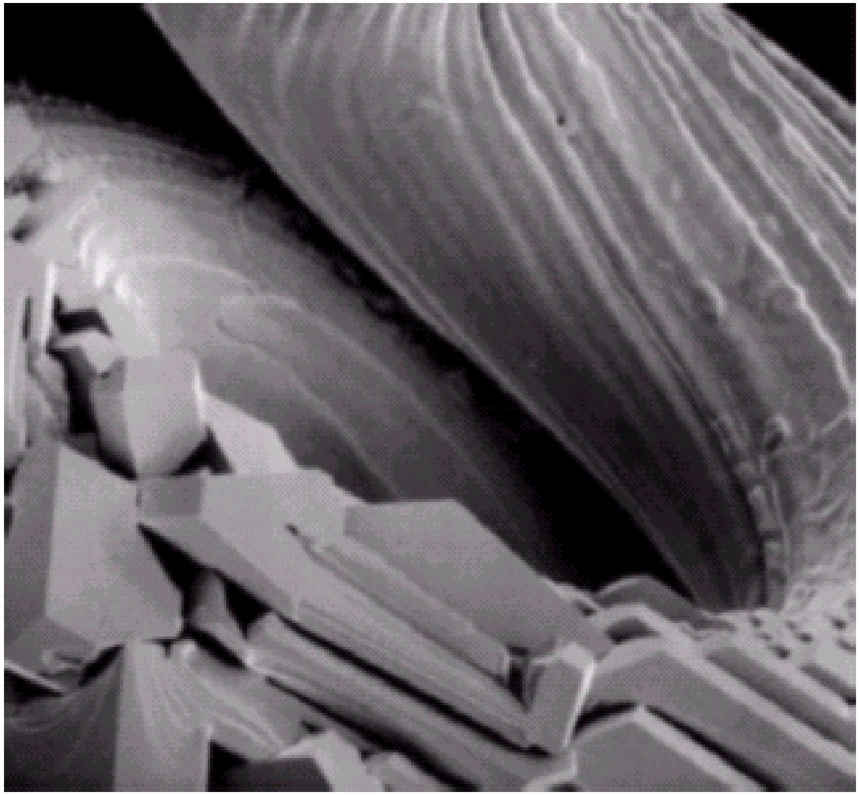


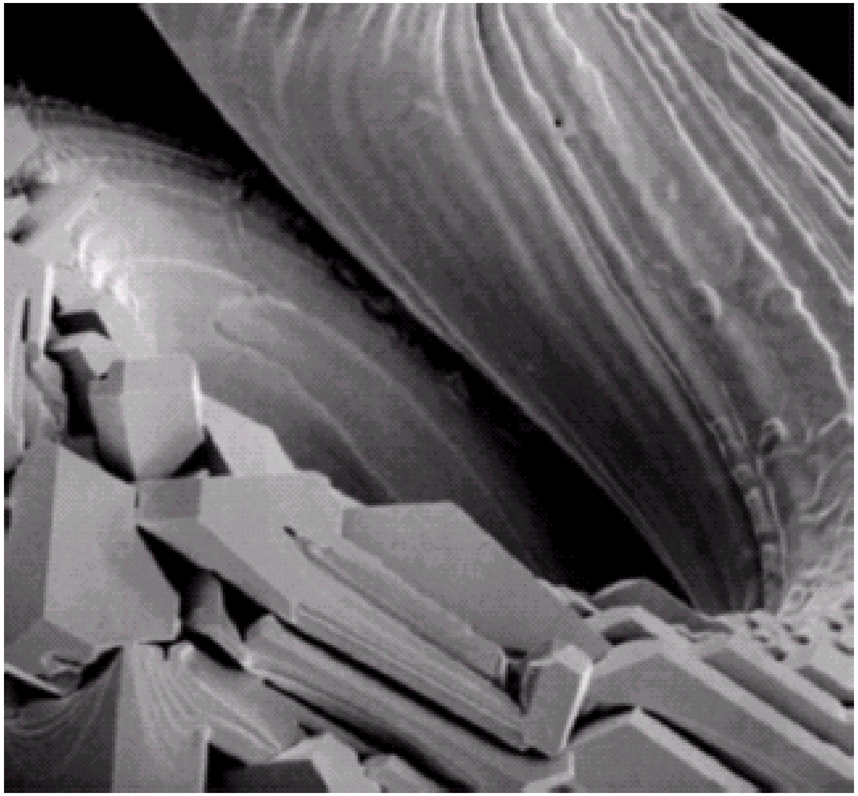
0	-1	0
-1	5	-1
0	-1	0



Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Simplified Image Enhancement (cont…)





Variants On The Simple Laplacian

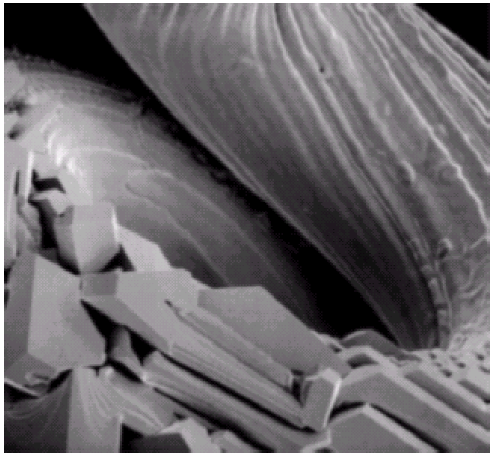
There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	-4	1
0	1	0

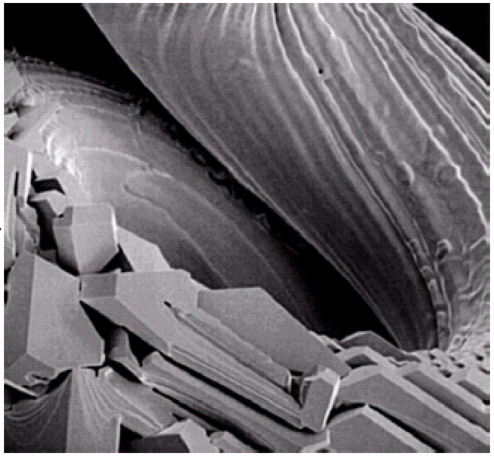
Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian



-1	-1	-1
-1	9	-1
-1	-1	-1



1st Derivative Filtering

Implementing 1st derivative filters is difficult in practice

For a function $f(x, y)$ the gradient of f at coordinates (x, y) is given as the column vector:

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

1st Derivative Filtering (cont…)

The magnitude of this vector is given by:

$$\begin{aligned}\nabla f &= mag(\nabla \mathrm{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}\end{aligned}$$

For practical reasons this can be simplified as:

$$\nabla f \approx |G_x| + |G_y|$$

1st Derivative Filtering (cont…)

There is some debate as to how best to calculate these gradients but we will use:

$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right|$$
$$+ \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

which is based on these coordinates

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Sobel Operators

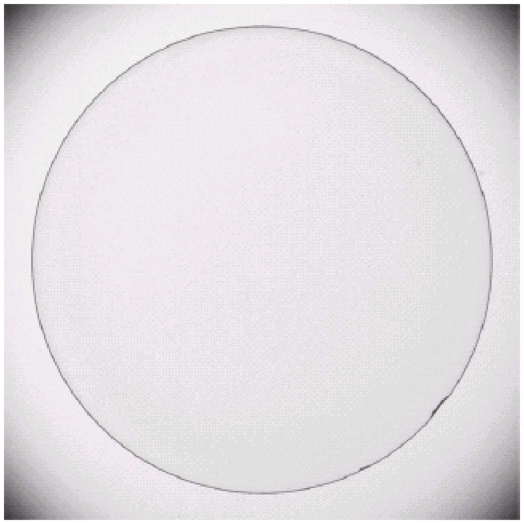
Based on the previous equations we can derive the *Sobel Operators*

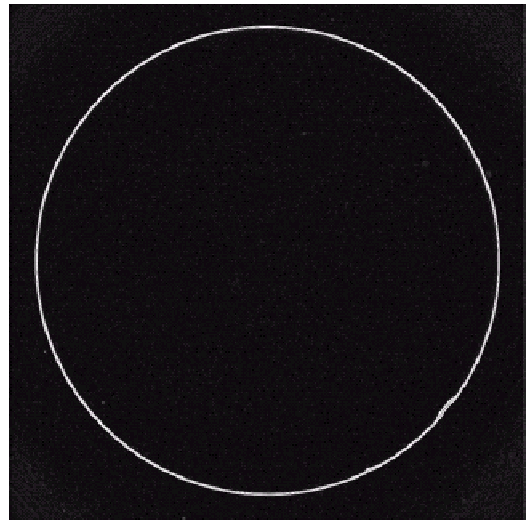
-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

To filter an image it is filtered using both operators the results of which are added together

Sobel Example





An image of a contact lens which is enhanced in order to make defects (at four and five o'clock in the image) more obvious

Sobel filters are typically used for edge detection

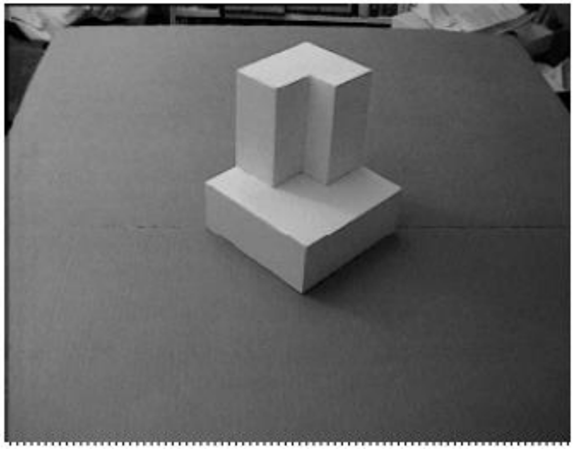
What is an Edge?

- Sharp change in brightness (discontinuities).
- Where do edges occur?
  - Actual edges: Boundaries between objects
  - Sharp change in brightness can also occur within object

    Reflectance changes

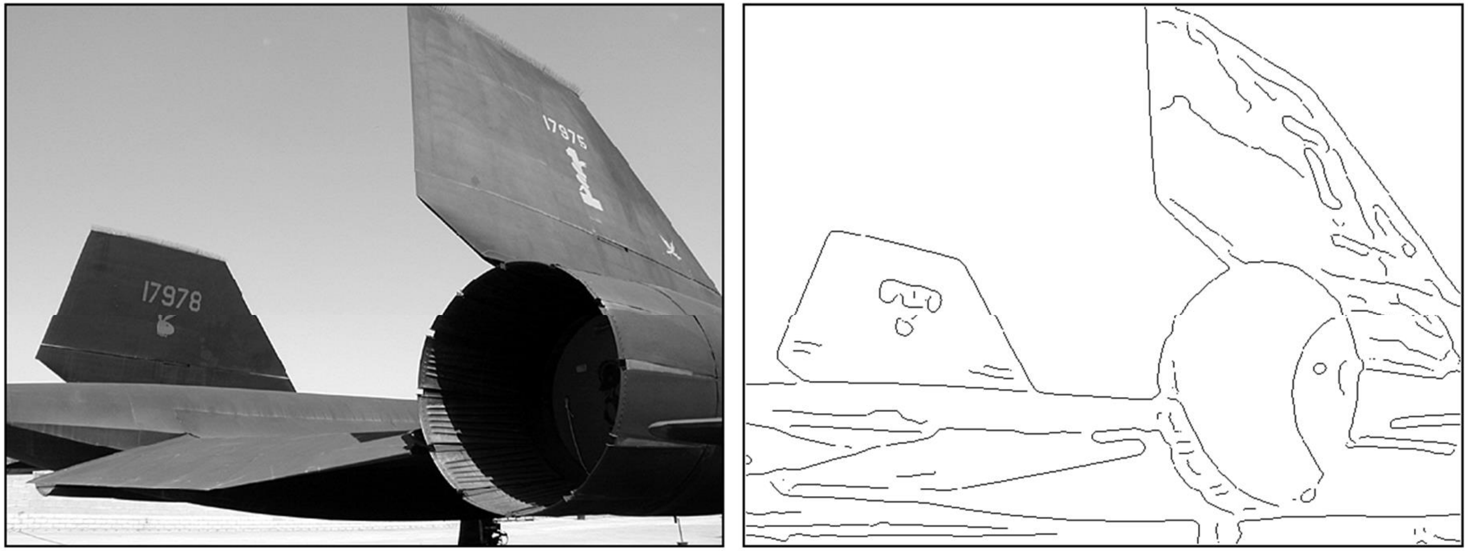
    Change in surface orientation

    Illumination changes. E.g. Cast shadow boundary



Edge Detection

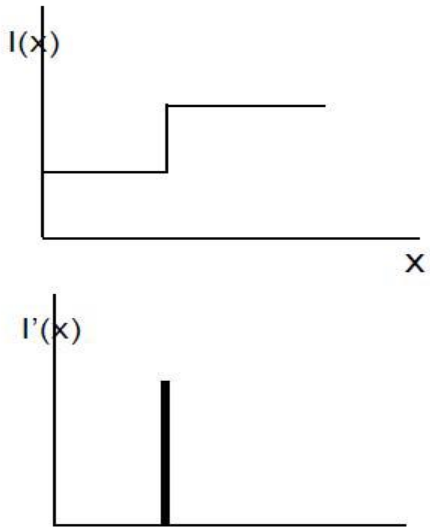
- Image processing task that finds edges and contours in images
- Edges so important that human vision can reconstruct edge lines

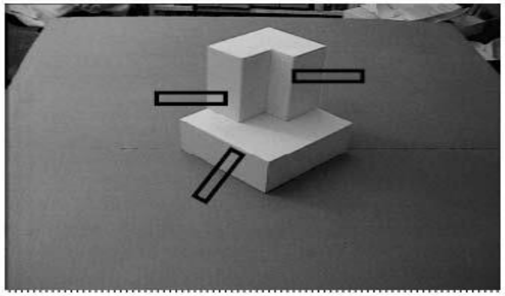


(a) (b)

Characteristics of an Edge

- Edge: A sharp change in brightness.
- Ideal edge is a step function in some direction.

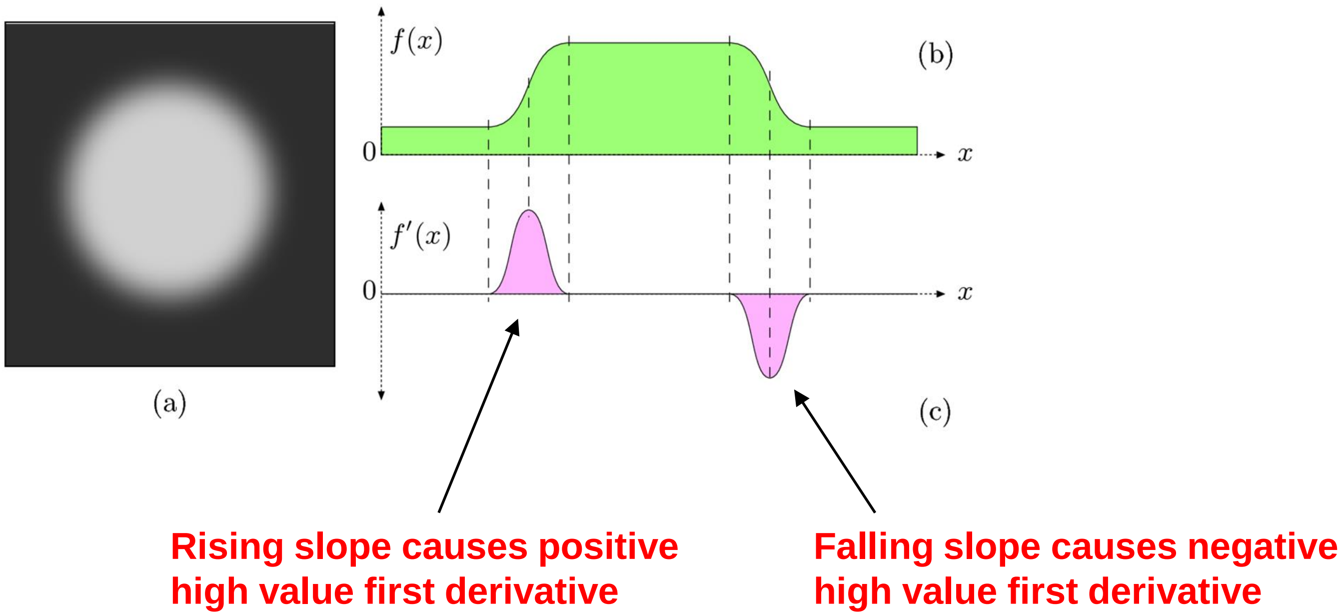




Characteristics of an Edge

- Real (non-ideal) edge is a slightly blurred step function.
- Edges can be characterized by high value first derivative.

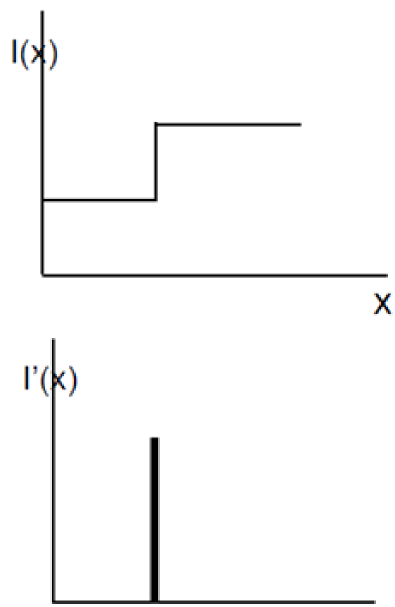
$$f'(x) = \frac{df}{dx}(x)$$

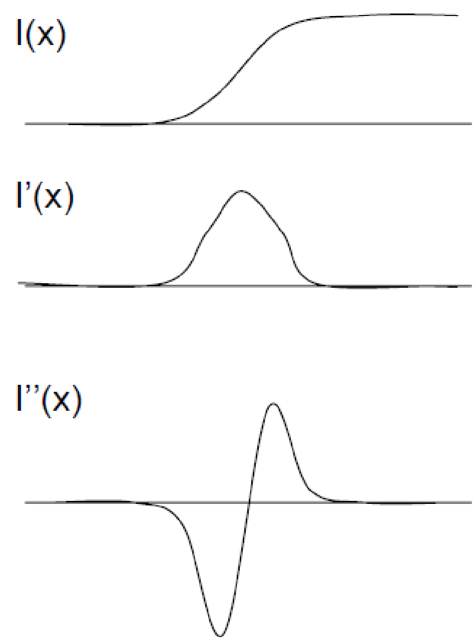


Characteristics of an Edge

- Ideal edge is a step function in certain direction.
- First derivative of I(x) has a **peak** at the edge
- Second derivative of I(x) has a **zero crossing** at edge

Ideal edge





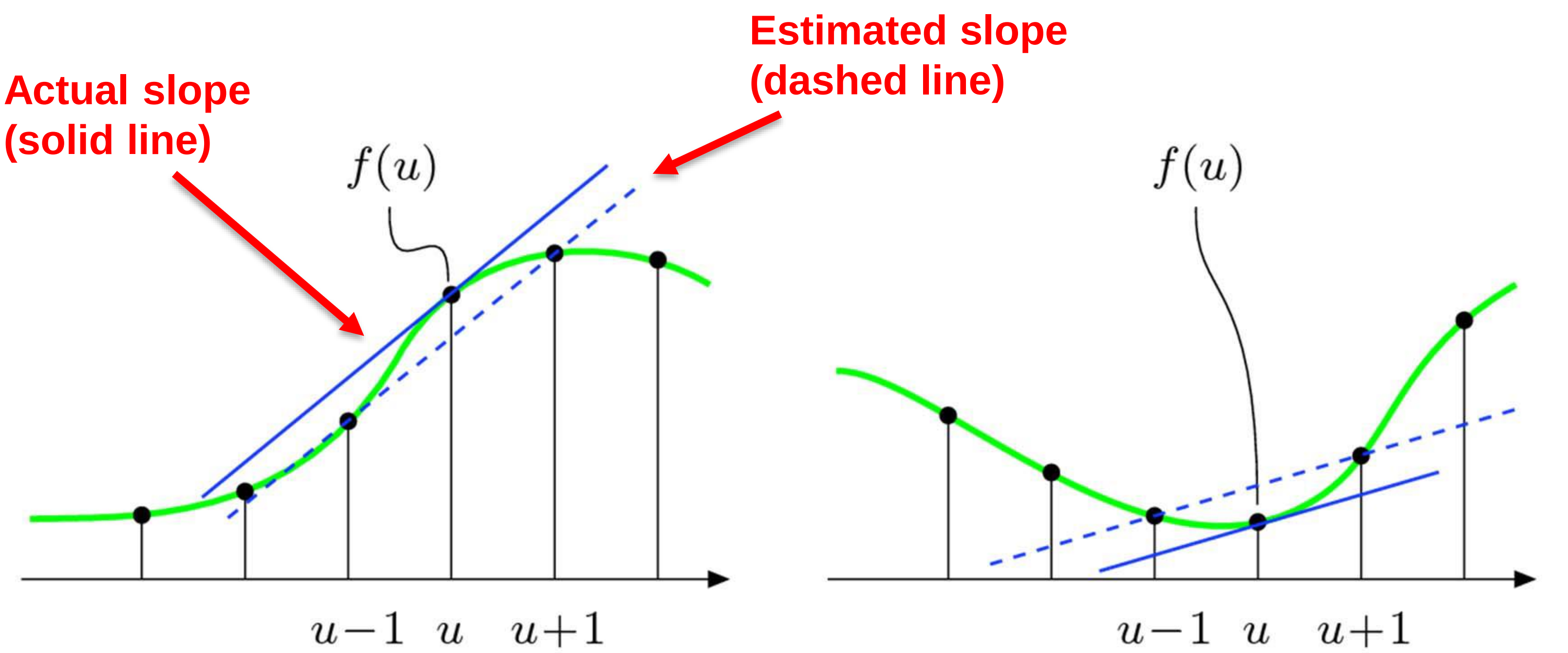
Real edge

First derivative shows peak

Second derivative shows zero crossing

Computing Derivative of Discrete Function

$$\frac{df}{du}(u) \approx \frac{f(u+1) - f(u-1)}{2} = 0.5 \cdot \big(f(u+1) - f(u-1)\big)$$



Finite Differences

- Forward difference (right slope)

$$\Delta_+ f(x) = f(x+1) - f(x)$$

- Backward difference (left slope)

$$\Delta_- f(x) = f(x) - f(x-1)$$

- Central Difference (average slope)

$$\Delta f(x) = \frac{1}{2}\left(f(x+1) - f(x-1)\right)$$

1^{st} & 2^{nd} Derivatives

Comparing the 1^{st} and 2^{nd} derivatives we can conclude the following:

- 1^{st} order derivatives generally produce thicker edges
- 2^{nd} order derivatives have a stronger response to fine detail e.g. thin lines
- 1^{st} order derivatives have stronger response to grey level step
- 2^{nd} order derivatives produce a double response at step changes in grey level

Summary

In this lecture we looked at:

- Sharpening filters
    - 1st derivative filters
    - 2nd derivative filters
- Combining filtering techniques

Combining Spatial Enhancement Methods

- Successful image enhancement is typically not achieved using a single operation
- Rather we combine a range of techniques in order to achieve a final result
- This example will focus on enhancing the bone scan to the right

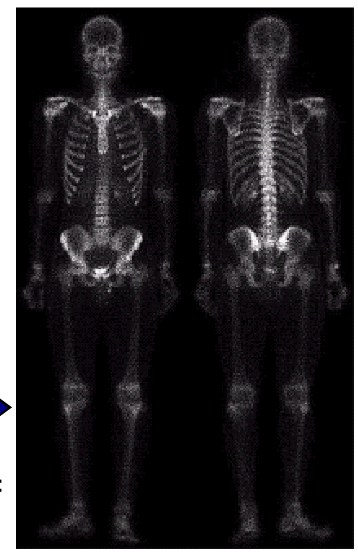
Combining Spatial Enhancement Methods (cont…)

(a)

Laplacian filter of bone scan (a)

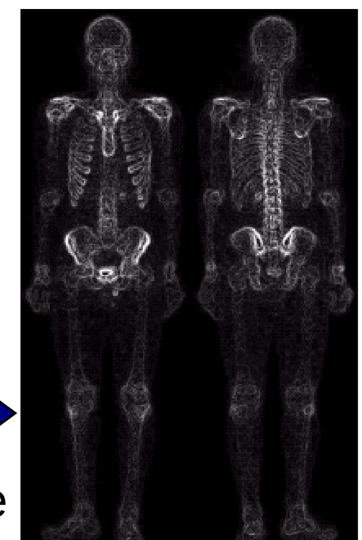
(b)

Sharpened version of bone scan achieved by subtracting (a) and (b)



(c)

Sobel filter of bone scan (a)



(d)

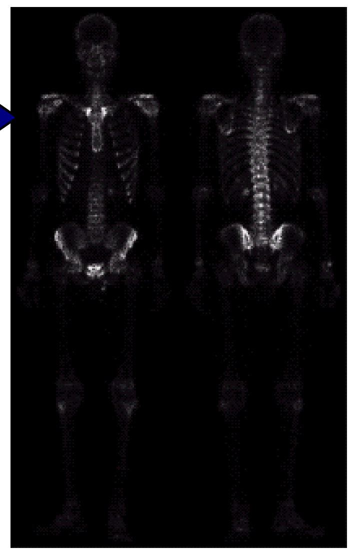
Combining Spatial Enhancement Methods (cont…)

Image (d) smoothed with a 5*5 averaging filter

(e)

The product of (c) and (e) which will be used as a mask

(f)

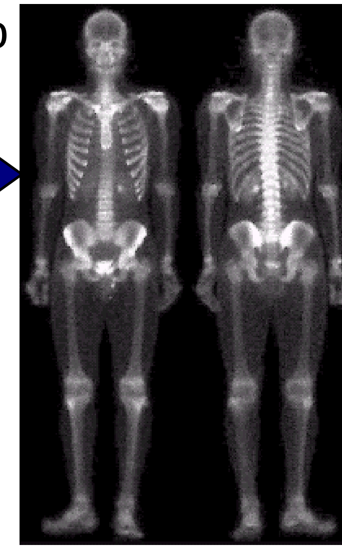


Sharpened image which is sum of (a) and (f)

(g)

Result of applying a power-law trans. to (g)

(h)



Combining Spatial Enhancement Methods (cont…)

Compare the original and final images